Rapid and slow decoherence in conjunction with dissipation in a system of two-level atoms

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We investigate the time evolution of a superposition of macroscopically distinct quantum states in a system of two-level atoms interacting with a thermal environment of photon modes. We show that the atomic coherent states are robust against decoherence, therefore we call their superpositions atomic Schrödinger cat states. The initial fast regime of the time evolution is associated with the process of decoherence, and it is directed towards the statistical mixture of the constituent coherent states of the original state for most of the initial conditions. However, certain superpositions, called symmetric, exhibit exceptionally slow decoherence. By introducing a new measure, we generalize the usual decoherence scheme regarding the evolution of the state to account also for the symmetric case. To stress the fact that the environment preserves symmetric superpositions much longer than the other ones, we present Wigner function images of the decoherence of a suitably oriented four component cat state.

PACS: 03.65.Bz, 03.67.L, 42.50.Fx

I. INTRODUCTION

The apparent lack of a superposition of macroscopically distinct quantum states (Schrödinger cats) has been an interesting and vivid problem since Schrödinger's famous paper [1]. A successful approach, initiated by Zeh [2] and developed by Zurek [3], obtains the loss of quantum coherence as the consequence of the unavoidable interaction with the environment. Theoretical studies in this framework have investigated a variety of model systems usually coupled to a collection of harmonic oscillators as an environment. Fundamental work has been done on this subject in [4–10], for reviews see [11,12]. Important experiments have been carried out during the last years [13–15] and the possible production of long-lived quantum superpositions has gained wide attention because of their expected use in quantum computation [16].

In the present paper we investigate a system which is a candidate for the experimental study of decoherence and possibly also for practical applications. The model to be described in detail in section II, consists of several identical two-level atoms (the system) interacting with a large number of photon modes in a thermal state (the environment). It has the advantage that it is simple to make the correct transition from a microscopic system to a macroscopic one by increasing the number of atoms. We analyze the evolution of the reduced density matrix of the atomic system by the appropriate master equation [17–19]. One of the main results of the present work is that we show how one and the same solution of this master equation describes both decoherence and dissipation.

By analytical short-time calculations we show in section III that the atomic coherent states [20] of our system are robust against decoherence caused by the realistic interaction we consider. Since this behavior justifies that the superpositions of atomic coherent states are relevant with respect to the original problem of Schrödinger, such a superposition is rightly called an atomic Schrödinger cat state [21–23]. We also note that there are several proposals for the experimental preparation of these type of states [24,25].

Next, in section IV we present the decoherence and dissipation properties of atomic Schrödinger cat states, based on numerical computations of their time evolution. The time scales of decoherence and dissipation differ by orders of magnitude. Using this fact, we show how one can make a clear distinction between these two processes despite of the interplay between them, and we define the characteristic time of decoherence. This decoherence time strongly depends on the initial conditions, notably, it is particularly large for a special set of initial cat states [22,26]. This will be termed as slow decoherence in contrast with the general case which will be referred to as rapid decoherence.

The interplay between decoherence and energy dissipation is the most appreciable in connection with the concept of pointer states [3]. It will be shown that when the decoherence is rapid, then the constituent coherent states of the initial state are pointer states to a very good approximation. However, when there is enough time for dissipation, i.e. when decoherence is slow, then the initial atomic coherent states themselves evolve into mixtures, and therefore a refined scheme of decoherence holds.

In order to underline the contrast between rapid and slow decoherence we superpose four atomic coherent states corresponding to the vertices of a suitably oriented tetrahedron. The time evolution of this four component cat state will be studied by the aid of the spherical Wigner function in section V. As it is expected, the interaction with the environment selects that pair from the initial superposition which constitutes a long-lived cat state.

II. DESCRIPTION OF THE MODEL

We consider a system of identical two-level atoms interacting with the environment of macroscopic number of photon modes. With dipole interaction and in the rotating wave approximation the total system is described by the following model Hamiltonian:

$$H = \hbar\omega_a J_z + \sum_k \hbar\omega_k a_k^\dagger a_k + \sum_k \hbar g_k \left(a_k^\dagger J_- + a_k J_+ \right), \quad (1)$$

where ω_a is the transition frequency between the two atomic energy levels, ω_k denote the frequencies of the modes of the environment and g_k are coupling constants. J_+, J_- and J_z are

dimensionless collective atomic operators obeying the usual angular momentum commutation relations [27]. This Hamiltonian leads to the interaction picture master equation for the reduced density matrix of the atomic system, ρ [17,18]:

$$\frac{d\rho(t)}{dt} = -\frac{\gamma}{2} \left(\langle n \rangle + 1\right) \left(J_+ J_- \rho(t) + \rho(t) J_+ J_- - 2 J_- \rho(t) J_+\right)$$
$$-\frac{\gamma}{2} \langle n \rangle \left(J_- J_+ \rho(t) + \rho(t) J_- J_+ - 2 J_+ \rho(t) J_-\right). \quad (2)$$

Here $\langle n \rangle$ is the mean number of photons in the environment and γ denotes the damping rate. Note that the same master equation can be obtained by considering a low-Q cavity containing Rydberg atoms [19].

If the state of the atomic system was initially invariant with respect to the permutations of the atoms, i.e. it was a superposition of the totally symmetric Dicke states [27], the dipole interaction described by $H_{int} = \sum_k \hbar g_k \left(a_k^\dagger J_- + a_k J_+\right)$ in (1) would not destroy this symmetry. Therefore we may restrict our investigation of the N atom system to the totally symmetric $N + 1$ dimensional subspace of the whole Hilbert-space of the atomic system. This subspace is isomorphic to an angular momentum eigensubspace labeled by $j = N/2$. This model has been proven to be valid in cavity QED experiments with many atoms, as reviewed in [19].

The environment as a static reservoir (represented by the thermal photon modes) continuously interacts with the atomic system influencing its dynamics. As it is obvious, the dissipation of the energy leads to thermal equilibrium in the system, corresponding to the stationary solution of the master equation (2). However, as it will be shown here, the same master equation describes also a much more interesting process. The continuous "monitoring" [3] of the atomic system by the environment results in the total loss of the coherence of the quantum superpositions in the system. This decoherence process is generally extremely fast compared to the dissipation, except for special initial conditions which will be discussed in subsection IV A.

III. THE INITIAL STAGE OF THE TIME EVOLUTION

In this section we apply general concepts to our system in order to find the initial states for the master equation (2) which are relevant to the original problem of Schrödinger [1] concerning the unobservability of macroscopic superpositions.

First we consider the short-time behavior of the total system. The (by assumption pure) system+environment state, $|\Psi\rangle$ can be written in the Schmidt representation [28–30,12] at any time as

$$|\Psi(t)\rangle = \sum_k \sqrt{p_k(t)} \, |\varphi_k(t)\rangle \, |\Phi_k(t)\rangle , \quad (3)$$

where $|\varphi_k\rangle$ and $|\Phi_k\rangle$ are elements of certain orthonormal bases (Schmidt bases) of the system and the environment, respectively.

At zero temperature the photon field of the present model is in its pure vacuum state $|0\rangle$, therefore the initial state factorizes as

$$|\Psi(0)\rangle = |\varphi_0(0)\rangle \, |0\rangle , \quad (4)$$

i.e. $p_0(0) = 1$ and $p_k(0) = 0$ for $k \neq 0$. Due to the interaction this product state evolves into a more general Schmidt sum like Eq. (3), or in other words it turns into an entangled state. The time scale of this entanglement or de-separation can be obtained [29] in leading order of time as

$$p_0(t) = 1 - At^2, \quad (5)$$

with

$$A = \sum_{k \neq 0, l \neq 0} |\langle \varphi_k(0)| \, \langle \Phi_l(0)| \, H_{int} \, |\varphi_0(0)\rangle \, |0\rangle|^2 . \quad (6)$$

This quantity can be called the rate of entanglement. Using the explicit form of the interaction Hamiltonian H_{int}, a straightforward calculation leads to

$$A = \mathcal{C}\left(J_+, J_-\right) := \langle J_+ J_- \rangle - \langle J_+ \rangle \langle J_- \rangle , \quad (7)$$

i.e. in our system the rate of entanglement is the normally ordered correlation function of the operators J_+ and J_-.

Let us turn to the case of finite temperatures, when the total system has to be represented by a mixed state even at $t = 0$. The linear entropy, defined as

$$S_{lin} = \text{Tr}(\rho - \rho^2), \quad (8)$$

can be regarded as a relevant measure of decoherence [12,31]. Restricting ourselves again to the initial regime of the time evolution, we can make use of the master equation (2) and calculate the time derivative of the linear entropy at $t = 0$:

$$\left(\frac{\partial S_{lin}}{\partial t}\right)_{t=0} = \gamma \left(\langle n \rangle \, \mathcal{C}\left(J_-, J_+\right) + \left(\langle n \rangle + 1\right) \mathcal{C}\left(J_+, J_-\right)\right). \quad (9)$$

The normally (antinormally) ordered correlation function, $\mathcal{C}\left(J_+, J_-\right)$ ($\mathcal{C}\left(J_-, J_+\right)$), disappears in the eigenstate $|j, m = -j\rangle$ ($|j, m = j\rangle$) of J_- (J_+). However, the collective atomic operators J_- and J_+ have no simultaneous eigenstates which would annullate the right hand side of Eq. (9). Nevertheless, we are going to show that if the number of atoms $N = j/2$ is large enough, then the correlation functions in Eq. (9) are negligible in a class of states called atomic coherent states [20]. These states are labeled by a complex parameter $\tau = \tan(\beta/2) \exp(-i\phi)$ (for the angles β and ϕ see Fig. 1) and can be expanded in terms of the eigenstates of the operator J_z (Dicke states) as

$$|\tau\rangle = \sum_{m=-j}^{j} \binom{2j}{j+m}^{\frac{1}{2}} \frac{\tau^{j+m}}{(1+|\tau|^2)^j} |j, m\rangle. \quad (10)$$

For large j, the atomic coherent states are approximate eigenstates of the operators J_- and J_+ [20,26]. This statement is understood in the sense that the square of the cosine of the angle α between $|\tau\rangle$ and $J_- |\tau\rangle$:

$$\cos^2 \alpha = \frac{|\langle \tau |J_-| \tau \rangle|^2}{\langle \tau \mid \tau \rangle \langle \tau |J_+ J_-| \tau \rangle} \quad (11)$$

differs from unity by a factor which scales as $(j\tau^2)^{-1}$. Thus α becomes negligible in the $j \to \infty$ limit for finite τ [26]. The same statement holds for the operator J_+, therefore both

correlation functions in Eq. (9) are indeed negligible in the atomic coherent states (10).

This suggests that the atomic coherent states are rather stable against the decoherence induced by the photon modes, i.e. they can serve as a model of classical-like macroscopic quantum states. This result is in analogy with the stability of the oscillator coherent states obtained in [31].

Two such states, $|\tau_1\rangle$ and $|\tau_2\rangle$ can be considered as macroscopically distinct, whenever the distance between the parameters τ_1 and τ_2 is sufficiently large on the complex plane. This implies that the coherent superposition of these states, which will be introduced in the next section, yields an appropriate model of the original paradox of Schrödinger.

IV. DECOHERENCE OF ATOMIC SCHRÖDINGER CATS

Based on the results of the previous section, the superpositions

$$|\Psi_{12}\rangle = \frac{|\tau_1\rangle + |\tau_2\rangle}{\sqrt{2(1 + \mathrm{Re}\,\langle\tau_1|\tau_2\rangle)}} \qquad (12)$$

will be called atomic Schrödinger cat states [21,22,32], see Fig. 1. Now we are going to present our results on the decoherence and dissipation dynamics of these type of states.
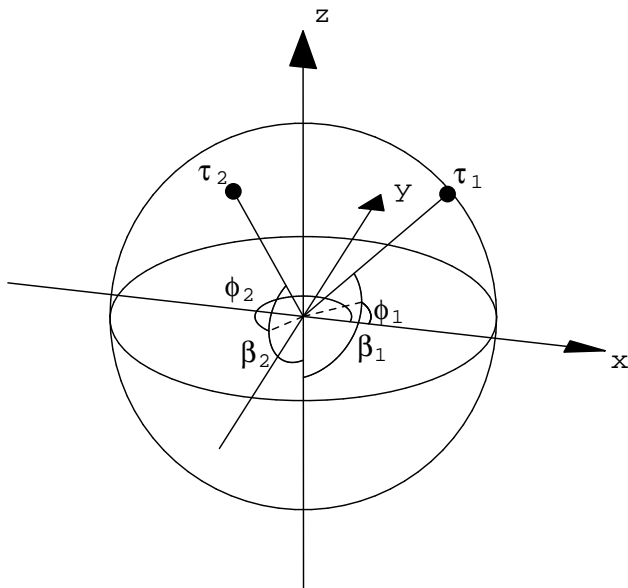

FIG. 1. Scheme of an atomic Schrödinger cat state defined by Eq. (12). The points labeled by τ_1 and τ_2 represent the corresponding atomic coherent states on the surface of the Bloch-sphere. The angles defining the τ parameters are also shown.

A. Time scales

A typical result of the numerical integration of Eq. (2) is that the time evolution of the states given by Eq. (12) can be characterized by two different time scales, as illustrated by Fig. 2, where the linear entropy and the energy of the atomic system is plotted versus time.
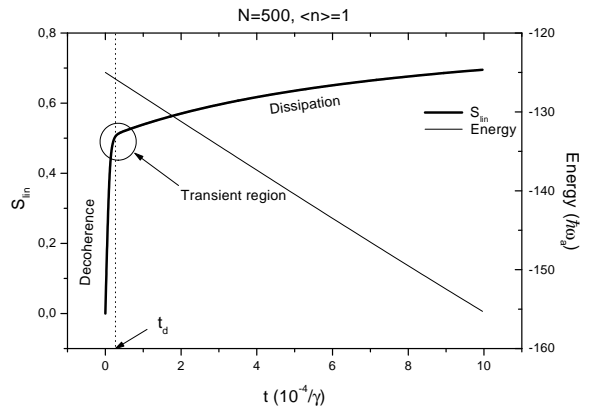

FIG. 2. The two regimes of the time evolution. (Initially: $\tau_1 = \tan\pi/4$, $\tau_2 = 0$.) The number of atoms is $N = 500$ and the average number of photons is $\langle n \rangle = 1$, $t_{\mathrm{d}} \approx 6 \times 10^{-5}/\gamma$

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As we can see, there exists a time instant t_{d} (marked with an arrow in Fig. 2) when the character of the physical process changes radically. Initially $S_{\mathrm{lin}}(t)$ increases rapidly while the dissipated energy of the atoms is just a small fraction of that part of the energy which will eventually be transferred to the environment. On the other hand, for longer times $t \gg t_{\mathrm{d}}$ both curves change on the same time scale. The energy of the atomic system decays exponentially as a function of time allowing for identifying the characteristic time of the dissipation t_{diss}, with the inverse of the exponent. (We note that in Fig. 2 the plotted time interval is much shorter than t_{diss}, thus the exponential behavior is not seen.) More detailed calculations have shown that for high temperatures energy and linear entropy exhibit similar exponential behavior in the second regime of the time evolution. Their exponents coincide with 2-3% relative error. This implies that the initial stage of the time evolution is dominated by decoherence while after t_{d} the dissipation determines the dynamics. Accordingly we define the characteristic time of the decoherence as the instant when the slope of the curve $S_{\mathrm{lin}}(t)$ decreases appreciably. We note that t_{d} defined in this way is in accordance with the decoherence time defined previously in [22] for a specific initial state.

It is remarkable that although a few hundred atoms do not really constitute a macroscopic system, the difference of the time scales is obviously seen in Fig. 2. It is generally true that the larger N is, the more naturally and sharply the time evolution splits into two regimes.

Now we turn to the investigation of the dependence of the decoherence time t_{d} on the initial conditions.
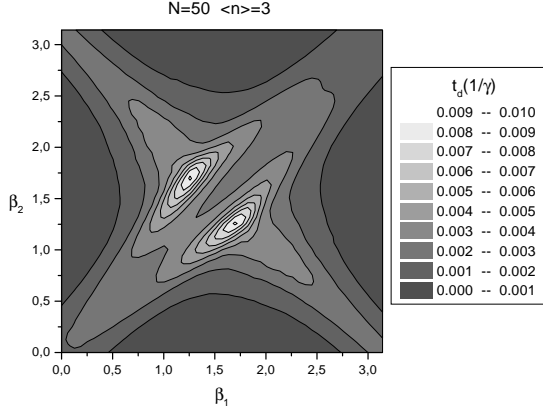
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FIG. 3. The dependence of the characteristic time of the decoherence on the parameters of the initial Schrödinger cat state: $\tau_1 = \tan \beta_1/2$, $\tau_2 = \tan \beta_2/2$. The number of atoms is $N = 50$ and the average number of photons is $\langle n \rangle = 3$.

Fig. 3 shows the contour plot of the decoherence time versus the parameters β_1 and β_2 (see Fig. 1) of the initial atomic Schrödinger cat state (12). We have set $\phi_1 = \phi_2 = 0$ for simplicity. As we can see, the effect of decoherence is remarkably slower when $\beta_1 \approx \beta_2$ which was expected since in this case the overlap of the two initial coherent states is not negligible, so these states can not be considered as "macroscopically distinct". Much more surprising is the fact that cat states which were initially symmetric with respect to the (x, y) plane (i. e. $\beta_1 \approx \pi - \beta_2$) also decohere slower [22,32], but it is in accordance with the analytical estimations of Braun et. al. [26] In the following sections we shall refer to these states as *symmetric* ones.

B. The direction of the decoherence

We saw in the previous subsection that the interplay between decoherence and dissipation is reflected in the time evolution of the superpositions given by Eq. (12). In this subsection we shall focus on the direction of the process resulting from the dynamics governed by the master equation (2).

According to the general theory of environment induced decoherence [3,11,12,31,33], the interaction with a large number of degrees of freedom selects naturally the so-called pointer basis [3] in the Hilbert-space of the system subject to decoherence. This process is claimed to favor the constituent states of the pointer basis in the sense that the system is driven towards a classical statistical mixture of these states. Thus, from the present point of view $\rho(t_d)$ is the relevant quantity to be examined.

Recalling the analytical results of sec. III, it seems plausible to expect that the atomic coherent states (10) will be pointer states.

By introducing the density matrix which corresponds to the classical statistical mixture of the initial coherent states:

$$\rho_{\rm cl}(\tau_1, \tau_2) = \frac{1}{2} \left(|\tau_1\rangle\langle\tau_1| + |\tau_2\rangle\langle\tau_2| \right), \qquad (13)$$

the expected scheme of the decoherence reads:

$$|\Psi_{12}\rangle\langle\Psi_{12}| \rightarrow \rho_{\rm cl}(\tau_1, \tau_2). \qquad (14)$$
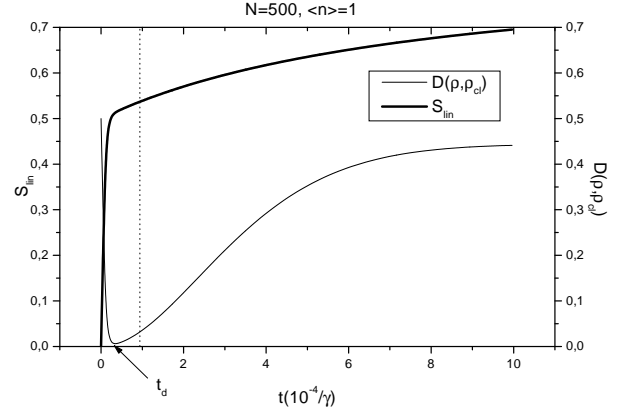
We shall refer to $\rho_{\rm cl}$ as the classical density matrix.

FIG. 4. The linear entropy $S_{\rm lin}$ and the distance D between ρ and $\rho_{\rm cl}$ (defined by Eq. (15)) in the case of a rapidly decohering Schrödinger cat state ($\tau_1 = \tan \pi/4$, $\tau_2 = 0$). The number of atoms is $N = 500$ and the average number of photons is $\langle n \rangle = 1$.

The distance between the actual density matrix $\rho(t)$ and $\rho_{\rm cl}$, defined with

$$D\left(\rho(t), \rho_{\rm cl}\right) = \mathrm{Tr}\left[(\rho(t) - \rho_{\rm cl})^2 \right], \qquad (15)$$

is always decreasing fast. Except for the case of slowly decohering cat states which will be discussed below, $D\left(\rho(t), \rho_{\rm cl}\right)$ reaches its minimal value at the *decoherence time*, see Fig. 4. This minimal value is very close to zero implying that the density matrix of the system at this instant is nearly the same as the classical density matrix (13). This fact justifies the definition of the characteristic time of the decoherence in section IV A, and it is in excellent agreement with the decoherence scheme (14).

Due to the exceptionally slow decoherence, we have to modify this picture if the initial state is a symmetric superposition. In this case the decoherence time is so long that the atomic coherent constituents of the initial state are also appreciably affected by the time evolution until the decoherence time, t_d. The state of the atomic system at t_d will be a mixture, which is the same as if the system had started from $\rho_{\rm cl}$ at t=0. In other words the evolution follows the modified scheme:

$$|\Psi_{12}\rangle\langle\Psi_{12}| \rightarrow \tilde{\rho}_{\rm cl}(\tau_1, \tau_2, t) \qquad (16)$$

where the time dependent classical density matrix $\tilde{\rho}_{\rm cl}(\tau_1, \tau_2, t)$ is the one which would evolve from the statistical mixture (13) $\rho_{\rm cl}(\tau_1, \tau_2) = \tilde{\rho}_{\rm cl}(\tau_1, \tau_2, t = 0)$ according to the same master equation (2) as the actual atomic density matrix. The distance between the time dependent classical density matrix, $\tilde{\rho}_{\rm cl}$ and $\rho(t)$ becomes negligible at t_d, and asymptotically reaches zero for long times in the case of all the initial conditions.

V. WIGNER FUNCTIONS OF FOUR COMPONENT SCHRÖDINGER CAT STATES

The results of the previous section have shown that both the characteristic time and the direction of the decoherence strongly depend on the initial conditions. In this section we illustrate this fact by tracking the decoherence of the superposition of four atomic coherent states:

$$|\Psi_{1234}\rangle = \frac{|\tau_1\rangle + |\tau_2\rangle + |\tau_3\rangle + |\tau_4\rangle}{\sqrt{2(2 + \mathrm{Re}\ \sum_{i>k}\langle\tau_i|\tau_k\rangle)}}. \qquad (17)$$

Since four points on the surface of a sphere are not distinguished with respect to each other if and only if they are the vertices of a regular tetrahedron inscribed in the sphere, we set the components of $|\Psi_{1234}\rangle$ according to this pattern. On the other hand, the z axis is distinguished in the present model because of the form of the Hamilton operator (1), therefore we orient the tetrahedron with one edge parallel to the z axis and the opposite edge parallel to the y axis, see Fig. 5. Although we have in principle two substantially different ways of considering the state represented by Fig. 5 as the superposition of *two* atomic Schrödinger cat states, according to the results of the previous section one expects that the environment naturally selects one of these possibilities via the different time evolutions: the quantum coherence between the components of the symmetric pair $|\Psi_{12}\rangle \propto |\tau_1\rangle + |\tau_2\rangle$ disappears slowly, while all the other pairs are rapidly decohering superpositions.
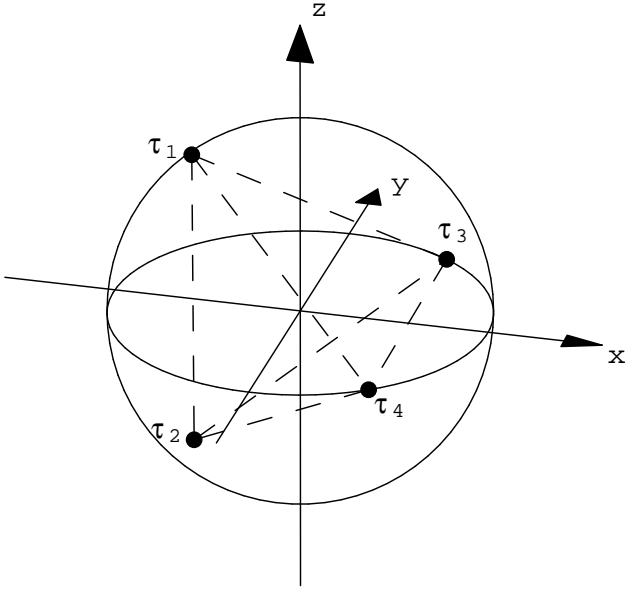

FIG. 5. Phase space scheme of the 4 component cat state. The atomic coherent states constituting the superposition (17) are represented by the points labeled by τ_1, ..., τ_4. They are arranged to form the vertices of a tetrahedron as shown.

We are going to visualize the decoherence process of $|\Psi_{1234}\rangle$ by the aid of the spherical Wigner function. It is a real func-

tion over the unit sphere (which is the appropriate phase space in the present case) being in a linear one-to-one correspondence with the density matrix of the atomic system.

Following the construction of Agarwal [34], the Wigner function corresponding to a reduced density matrix ρ of the atomic system is defined as the expectation value of the operator kernel $\Delta(\theta,\phi)$:

$$W(\theta,\phi) = \mathrm{Tr}\left(\rho\Delta(\theta,\phi)\right), \qquad (18)$$

with

$$\Delta(\theta,\phi) = \sum_{K=0}^{N}\sum_{Q=-K}^{K} T_{KQ}^{\dagger}Y_{KQ}(\theta,\phi). \qquad (19)$$

Here the T_{KQ} are the multipole operators and Y_{KQ} denote the spherical harmonics [35]. For previous applications of the spherical Wigner function see [21,22,36–39].

The Wigner function (18) suggestively maps the time evolution of the state (17) onto the unit sphere, as shown in Fig. 6. Here we present the state of the atomic system at three time instants, and we plot the corresponding Wigner functions both as a polar plot and as a contour plot. Dark shades mean negative, light shades mean positive function values. The four positive lobes, pointing from the center to the vertices of the tetrahedron shown in Fig. 5, correspond to the four atomic coherent states in (17). Due to the dissipation all these lobes will move slowly downwards. The initial interference pattern (Figs. 6(a) and (b)) has the regularity of the tetrahedron, there are equally pronounced oscillations along all the edges, representing the quantum coherence between the coherent states.

Figs. 6(c) and (d) depict the situation after a time which is short in the sense that the the shapes of the lobes of the coherent states are not appreciably affected (no dissipation), but the interference is already negligible between them, except for the single pair along the vertical edge of the tetrahedron. As it is seen from Fig. 5 this is the pair which represented initially the *symmetric* atomic Schrödinger cat state $\propto |\tau_1\rangle + |\tau_2\rangle$ in (17). The coherence between the components of this pair of states is nearly unaffected as shown by the strong oscillations.

A qualitatively different stage of the time evolution is shown in Figs. 6(e) and (f) at a later time. The coherent constituents are already affected by the dissipation, the uppermost one rather strongly, but the quantum coherence between the components of the symmetric pair is still present. On the contrary, the interference between all the other components has already disappeared.
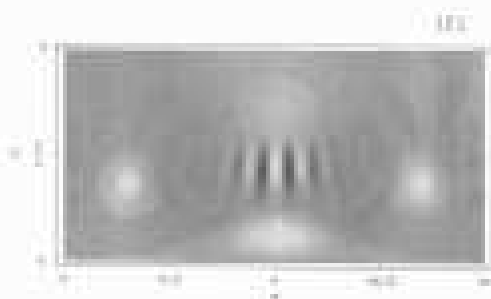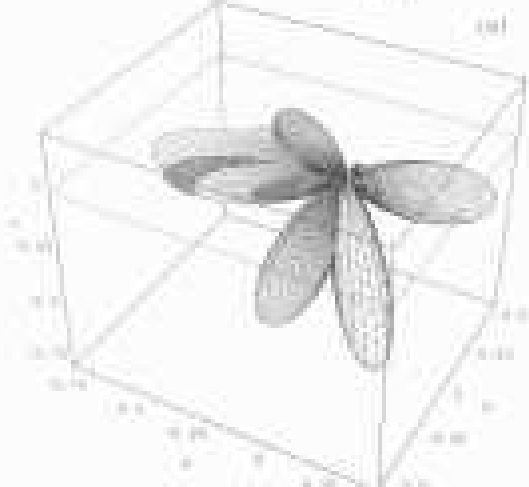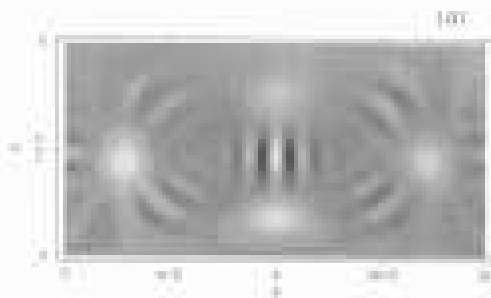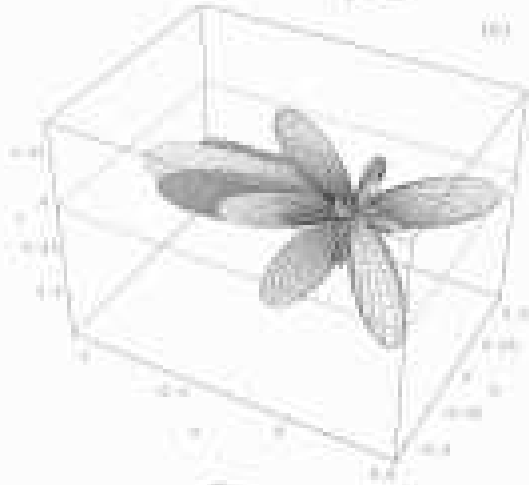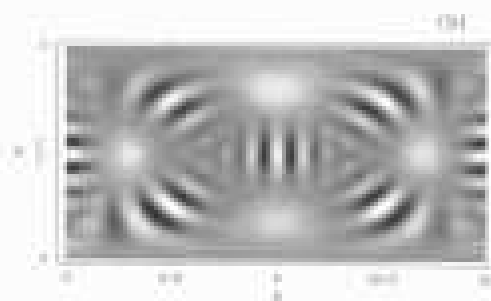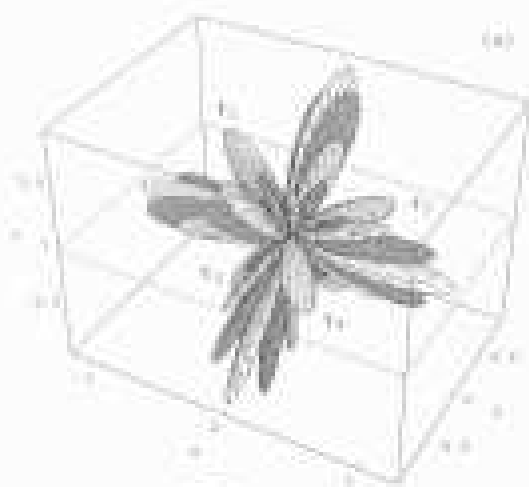
FIG. 6. Wigner view of the decoherence of the 4 component cat state. We plot the spherical Wigner function (18) both as a polar plot (a, c, e) and as a contour plot (b, d, f). A polar plot is obtained by measuring the absolute value of the function in the corresponding direction, and the resulting surface is shown in light where the values of the Wigner function are positive, and in dark where they are negative. Similarly, light shades of the contour plot correspond to positive function values, while dark shades mean negative function values. Plots (a, b) show the initial state, i.e. the superpositon $|\Psi_{1234}\rangle$ (Eq. (17)) as shown in Fig. 5. In (a) we also label the lobes corresponding to the initial coherent constituents. Plots (c, d) show the spherical Wigner function at $t = 0.015/\gamma$, while plots (e, f) correspond to $t = 0.04/\gamma$.

In view of the present results, if the initial state of the atomic system is a superposition of coherent states so that there are symmetric pairs of coherent states in the expansion, then the coherence between the components of these symmetric pairs will survive much longer than between any other terms.

VI. CONCLUSIONS

We have investigated the decoherence of superpositions of macroscopically distinct quantum states in a system of two-level atoms embedded in the environment of thermal photon modes. Utilizing the Schmidt decomposition and the linear entropy, we have shown that atomic coherent states are robust against decoherence, both at zero and non-zero temperatures. This result is in analogy with the harmonic oscillator case and justifies the definition of atomic Schrödinger cat states as superpositions of atomic coherent states.

By solving the master equation (2) we have identified two different regimes of the time evolution with the help of the linear entropy. The first one is dominated by decoherence while the second one is governed by dissipation. Based on several computational runs focusing on the characteristic times it was found that t_d decreases much faster than t_{diss} as the function of the number of atoms, N. Consequently t_d becomes many orders of magnitude smaller than the characteristic time of dissipation for macroscopical samples, and even for e.g. $N = 500$ atoms and an average photon number $\langle n \rangle = 1$ the ratio t_{diss}/t_d is around a few hundred depending on the initial conditions. However, there are very important exceptional cases, called slow decoherence, when the atomic coherent states constituting the initial atomic Schrödinger cat state are symmetric with respect to the equator of the Bloch sphere.

Using a new measure D, we have shown that at the characteristic time of decoherence the system is always very close to the state described by the time dependent classical density matrix. Apart from the exceptional case of slow decoherence, the coherent states appearing in (12) are approximate pointer states. When due to its symmetry the initial cat state is a long-lived superposition, also its constituent coherent states have time to transform into mixtures until t_d. We have given a modified scheme of decoherence which is valid also for slow decoherence. Accordingly, the orthonormal pointer basis consists of the eigenstates of $\tilde{\rho}_{cl}(\tau_1, \tau_2, t_d)$.

We have demonstrated the important difference between rapid and slow decoherence by tracking the time evolution of a four component superposition with the help of the spherical Wigner function. The initial interference pattern having the symmetry of a tetrahedron rapidly disappears except for the single slowly decohering pair.

ACKNOWLEDGMENTS

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